AIAA Journal
Vol. 48, No. 10, October 2010

Symmetrization Procedure for Structure–Fluid Cavity System Involving Pseudostatic Correction

A. Alia* and M. Souli†
Université des Sciences et Technologie de Lille, 59655 Villeneuve d'Ascq, France

DOI: 10.2514/1.J050056

This paper deals with a pressure-displacement formulation based on the finite-element method of an elastoacoustic coupling problem. Since directly solving the resulting system is CPU intensive for large models, the solution is usually based on only a few uncoupled structural modes in vacuum and rigid cavity modes. However this classical pressure-displacement formulation leads to nonsymmetric eigenvalue problems. Furthermore, if this method is accurate for weak coupling models, significant errors occur when the truncated modes are coupled to the ones used in the analysis. The present paper suggests a method combining the pseudostatic correction of the truncated modes in modal analysis with a symmetrization technique of the eigenvalue problems. It allows the calculation of the real coupled modes and frequencies, as well as the computation of an accurate elastoacoustic response, even for the strong coupling of a structure with a heavy fluid (liquid). It is less CPU expensive than the classical method, because it leads to a diagonalized system.

Nomenclature

A_f, A_s	=	compact forms of the matrices corresponding to fluid and structure, respectively
B	=	coupling matrix
C	=	compact form of the coupling matrix
c	=	sound speed
D_f, D_s	=	fluid and structure diagonal matrices containing the eigenvalues
F_f, F_s	=	fluid and structure loads
$F_{f\text{add}}F_f$, $F_{s\text{add}}F_s$	=	added loads due to the neglected cavity and structure modes, respectively
F_t	=	compact form of the load of the fluid
K_f	=	stiffness matrix for the fluid
K_s	=	structural stiffness matrix
k	=	wave number
M_f	=	mass matrix for the fluid
M_s	=	structural mass matrix
$M_{f\text{add}}$	=	added compressibility due to the neglected structure modes
$M_{s\text{add}}$	=	added mass due to the neglected cavity modes
N_f, N_s	=	fluid and structure shape functions
$\mathbf{n}$	=	normal
p, $\langle p^2 \rangle$	=	cavity pressure and cavity mean-square pressure
p_n	=	pressure due to truncated modes
S_t	=	compact form of the load of the structure
S^L, S^R	=	left and right transformation matrices
u	=	structure displacement
u_n	=	displacement due to truncated modes
V_η^L, V_η^R	=	left and right eigenvectors of the transformed system
V^L, V^R	=	left and right eigenvectors of the original system
$\langle v^2 \rangle$	=	structure mean-square velocity
X_f, X_s	=	fluid and structure eigenvectors
λ_f, λ_s	=	fluid and structure eigenvalues
ξ_f, ξ_s	=	modal pressure and modal displacement
ρ_f, ρ_s	=	fluid and structure densities
σ	=	stress
Γ_f, Γ_s, Γ_{sf}	=	fluid surface, structure surface, and fluid–structure surface
Γ_K, Γ_M	=	compact form for the coupled system
ϕ_s, ϕ_f	=	some fluid and some structure eigenvectors
Ω	=	diagonal matrix containing the eigenvalues of the coupled system
Ω_f, Ω_s	=	fluid and structure volumes
ω	=	pulsation

Presented as Paper 2009-4201 at the 39th AIAA Fluid Dynamics Conference, San Antonio, TX, 22–25 June 2009; received 10 July 2009; revision received 1 June 2010; accepted for publication 20 June 2010.

*Assistant Professor, Laboratoire de Mécanique de Lille, Université de Lille1; ahlem.alia@univ-lille1.fr.

†Professor, Laboratoire de Mécanique de Lille, Université de Lille1; mhamed.souli@univ-lille1.fr.

I. Introduction

IN THE last two decades, the noise radiated by vibrating structures has attracted increasing interest in many industrial applications such as aerospace and aeronautics. Vibroacoustics now represents a crucial aspect in the design of space vehicles, airplanes, and helicopters.

Typical examples include cabin noise inside vehicles and aircraft, which are always considered as a cavity with a flexible vibrating structure [1] and simulated by modal analysis [1–4]. However, modal analysis must be used with caution to obtain accurate results.

In the past, a large amount of effort has been devoted to investigate the vibroacoustic behavior of such systems. Stavrinidis et al. [2] gave an overview of approaches that have been used for the prediction of structural loads in satellites due to the acoustic environment. Pirk et al. [3] carried out the numerical vibroacoustic analysis of the fairing structure in the Brazilian vehicle satellite launcher. A cylindrical cavity was designed to represent a typical aircraft construction in the work of Pritchard et al. [4]. Modal tests were conducted for several configurations of the cylinder assembly under different load conditions in order to study the vibroacoustic response. Jollet et al. [5] used numerical approaches in the early stages of the satellite design to estimate the vibration environment of sensitive instruments and equipments.

In recent years, many numerical methods have been proposed to simulate the interaction between an elastic structure and fluid. This coupling can be done by using either coupled or uncoupled approaches that are based on different numerical methods, such as the boundary element method (BEM) [6–8], the finite-difference method (FDM) [9], and the finite-element method (FEM) [8,10–15].

For many problems, it is not necessary to consider a strong coupling. For instance, in the case of vehicles, many authors have treated the vibroacoustic coupling as a weak one [13]. In this approach, the mechanical response of the structure alone is calculated first. The obtained results are used as boundary conditions for the acoustic part of the problem. Hence, the vibrations of the structure are never influenced by the propagating waves through the fluid.

In fact, in their study of the elastoacoustic interaction, Kim et al. [16,17] have proposed an optimization procedure of real vehicle compartments by assuming that the acoustic pressure does not affect the structure behavior. They simulated the fluid by the BEM and the structure by the FEM. This optimization allowed them to reduce the thickness and the weight of the vehicle, as well as the internal acoustic pressure. The uncoupled approach was also used to predict the acoustic pressure of impacted plates by rigid bodies [18]. By assuming a weak coupling, Lambourg et al. [19] and Schedin et al. [20] have simulated the plate by the FDM and the fluid by the Rayleigh method.

It is obvious that the uncoupled method is easier to implement than the coupled one, since two small models are to be simulated one after the other [13]. However, this method is still limited to interaction problems involving heavy structures and light fluids. For instance, for underwater acoustics, the mechanical response is strongly affected by the surrounding fluid. In this case, it is necessary to study fluid–structure interaction problems in a coupled manner by considering the compressibility of the fluid and the structure flexibility effects.

In the coupled approach, the structure is always expressed in terms of a displacement-based formulation, while multiple choices of independent variables exist for the fluid [15]. When the fluid domain is represented by displacement potential, it leads to a symmetrical system. Expressing the fluid in terms of displacement involves a symmetric system and makes the coupling of their elements to the structural ones easy. However, this approach significantly increases the degrees of freedom of the fluid and, consequently, the computational time [14]. Another alternative involves the velocity potential formulation. Although the system is symmetric, it presents a nonclassical form of an eigenvalue problem [14,21]. Finally, the classical coupling based on pressure formulation also leads to a nonsymmetric eigenvalue problem.

The direct solution of these problems based on the FEM is computationally expensive for large-scale problems. Modal approaches are often preferred [22]. They constitute an efficient alternative when the problem size becomes more and more important. In the pressure-displacement-based formulation, the modal analysis consists of calculating only a few structure modes in vacuum and a few rigid cavity modes. However, this truncation can lead to significant errors for some strongly coupled vibroacoustic problems [22]. To improve the convergence, Tournour and Atalla [22] extended the pseudostatic correction (PSC), usually applied to elastic structures in a vibroacoustic interaction problem. They corrected the elastoacoustic response with a static correction in the classical modal superposition technique. However, they never computed the coupled modes of a nonsymmetric eigenvalue problem.

This paper adopts the pseudostatic method [22]. Left and right eigenvectors are calculated from a new symmetric form of the eigenvalue problem to transform the governing matrix system into a diagonal form. Sandberg [15] was the first to use this procedure to make the classical coupling eigenvalue problem symmetric without any static correction. This efficient and fast computational symmetrization method with PSC permits us to compute accurate modes, even for a strong coupling of a structure with a liquid.

The governing equations for vibroacoustic problems are first presented. Then, a review of the classical modal analysis is discussed. Finally, the PSC is described along with the different steps of the symmetrization procedure and validations of the method.

II. Statement of the Problem

The vibroacoustic problem in the present paper is the interaction of an elastic structure (Ω_s) with an ideal fluid occupying the volume (Ω_f) [Fig. 1]. A force is imposed on the surface (Γ_s). The boundary (Γ_f) is assumed to be rigid, whereas the surface (Γ_{sf}) represents the fluid–structure interface. The external normal of the structure is $\mathbf{n}_s$, and $\mathbf{n}$ is the external normal of the fluid cavity.

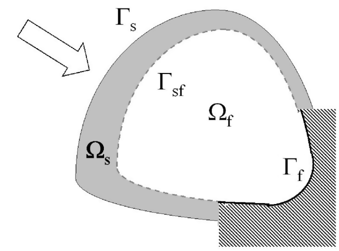

Fig. 1 Fluid–structure problem.

For a time-harmonic wave, the sound pressure P at any point fluctuates with a pulsation ω as

$$P = p\, e^{i\omega t} \tag{1}$$

where p is the pressure amplitude.

The wave equation in the case of a compressible and inviscid fluid is the following Helmholtz equation [23]:

$$\Delta p + k^2 p = 0; \qquad (\Omega_f) \tag{2}$$

where $k = \omega/c$ is the wave number, c is the sound velocity in the fluid, and ω is the circular frequency of the pressure oscillations. The normal derivative of the pressure is related to the normal velocity via the Euler equation $\frac{\partial p}{\partial n} = -i\rho\omega v_n$ for which the normal velocity is imposed on the surface. Since the surface (Γ_f) is rigid, the boundary condition for the fluid is

$$\frac{\partial p}{\partial n} = 0; \qquad (\Gamma_f) \tag{3}$$

The structure is governed by the elastic wave equation [21]:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = -\rho_s \omega^2 u_i + f_{vi}; \qquad (\Omega_s) \tag{4}$$

where σ is the stress, ρ_s is the structure density, u_i is the displacement in the ith direction, and f_{vi} is the mechanical volume load. By noting (Γ_s) the boundary on which a force is applied, the associated boundary condition is written as

$$\sigma_{ij} n_j^s = f_i; \qquad (\Gamma_s) \tag{5}$$

At the interface fluid–structure (Γ_{sf}), the continuity of the normal displacement and the stress gives

$$\frac{\partial p}{\partial n} = \rho_f \omega^2 \mathbf{u}\cdot\mathbf{n}; \qquad (\Gamma_{sf}); \qquad \sigma_{ij} n_j^s = -p n_i^s = p n_i; \qquad (\Gamma_{sf}) \tag{6}$$

III. Modal Analysis

A. Governing Equations

The application of the FEM to the variational formulation leads to the following linear system:

$$\begin{bmatrix} K_s - \omega^2 M_s & -B \\ -\omega^2 B^T & (K_f - \omega^2 M_f) \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ F_f \end{Bmatrix} \tag{7}$$

where

$$K_s = \int (\nabla N^s)^T D \nabla N^s \, \mathrm{d}\Omega_s$$

is the structural stiffness matrix,

$$M_s = \rho_s \int (N^s)^T N^s \, \mathrm{d}\Omega_s$$

is the structural mass matrix,

$$K_f = \frac{1}{\rho_f} \int (\nabla N^f)^T \nabla N^f \, \mathrm{d}\Omega_f$$

is the stiffness matrix for the fluid,

$$M_f = \frac{1}{\rho_f c^2} \int (N^f)^T N^f \, \mathrm{d}\Omega_f$$

is the mass matrix for the fluid, and

$$B = \int (N^f)^T n N^s \, \mathrm{d}\Gamma_{sf}$$

is the coupling matrix.

D in the expression of K_s is a symmetric matrix containing the structural elastic coefficients, N^s and N^f are the shape functions for structure and fluid, respectively. Equation (7) can be written in a condensed form as

$$(K - \omega^2 M)\begin{Bmatrix} u \\ p \end{Bmatrix} = F \tag{8}$$

Directly solving the linear system is a computationally inefficient approach because of the great number of DOFs in the coupled models. As mentioned previously, modal analysis significantly reduces the discretized problem size, and it allows us to diagonalize, block by block, the matrix system. The following generalized eigenvalue problems corresponding to the structure in vacuum and the fluid with stiff boundaries are independently solved:

$$K_s X_s = \lambda_s M_s X_s; \qquad K_f X_f = \lambda_f M_f X_f \tag{9}$$

where λ_s and λ_f are the structural and fluid eigenvalues, and χ_s and χ_f are the structural and fluid eigenvectors.

The matrices consisting of some eigenvectors in the structural and the fluid domains are ϕ_s and ϕ_f, respectively, such as

$$\begin{pmatrix} u \\ p \end{pmatrix} = \begin{bmatrix} \phi_s & 0 \\ 0 & \phi_f \end{bmatrix} \begin{pmatrix} \xi_s \\ \xi_f \end{pmatrix} \tag{10}$$

By using the following relation of orthogonalization,

$$\phi_s^T M_s \phi_s = I_s; \qquad \phi_s^T K_s \phi_s = D_s; \qquad \phi_f^T M_f \phi_f = I_f$$
$$\phi_f^T K_f \phi_f = D_f \tag{11}$$

the coupling system can be rewritten as follows:

$$\begin{bmatrix} D_s - \omega^2 I_s & -\phi_s^T B \phi_f \\ -\omega^2 \phi_f^T B^T \phi_s & D_f - \omega^2 I_f \end{bmatrix} \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{Bmatrix} \phi_s^T F_s \\ \phi_f^T F_f \end{Bmatrix} \tag{12}$$

It can take also the form,

$$\begin{bmatrix} D_s & -\phi_s^T B \phi_f \\ 0 & D_f \end{bmatrix} \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} - \omega^2 \begin{bmatrix} I_s & 0 \\ \phi_f^T B^T \phi_s & I_f \end{bmatrix} \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{Bmatrix} \phi_s^T F_s \\ \phi_f^T F_f \end{Bmatrix} \tag{13}$$

where D_s and D_f are diagonal matrices containing the structural and fluid eigenvalues, respectively, and ξ_s and ξ_f represent the modal structural displacement and the modal fluid pressure, respectively.

The resolution of the last algebraic linear system [Eq. (12)] for each frequency permits us to obtain the modal unknowns ξ_s and ξ_f and, consequently, u and p can be deduced from Eq. (10). Although its size is reduced, the system is a nonsymmetric eigenvalue problem [Eq. (13)]. Hence, efficient algorithms solving symmetric eigenvalue problems cannot be used. Many alternatives [15] have been proposed in order to diagonalize the matrix by considering the projection of the system on the left and right eigenvectors. By using some matrices transformations, Sandberg [15] wrote the right [Eq. (14)] and the left [Eq. (15)] eigenvalue problems into a symmetric form that permits us to diagonalize the system and solve the problem in an efficient manner. The direct method (DM) is compared with the modal approach and the Sandberg method in the next section. The following simulations are performed by using MATLAB on a PC (Intel Xeon, 4 GB, 3 GHz):

$$-\omega^2 \begin{bmatrix} I_s & 0 \\ 0 & I_f \end{bmatrix} \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} + \begin{bmatrix} D_s & -\sqrt{\rho c^2}\sqrt{D_s}\phi_s^T B \phi_f \\ -\sqrt{\rho c^2}\phi_f^T B^T \phi_s \sqrt{D_s} & D_f + \rho c^2 \phi_f^T B^T \phi_s \phi_s^T B \phi_f \end{bmatrix} \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = \begin{Bmatrix} \sqrt{\rho c^2}\sqrt{D_s}\phi_s^T F_s \\ \phi_f^T F_f - \rho c^2 \phi_f^T B^T \phi_s \phi_s^T F_s \end{Bmatrix} \tag{14}$$

where $\xi_s = (\sqrt{\rho c^2}\sqrt{D_s})^{-1}\eta_s$, $\xi_f = \eta_f$

$$-\omega^2 \begin{bmatrix} I_s & 0 \\ 0 & I_f \end{bmatrix} \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = \begin{bmatrix} D_s + \rho c^2 \phi_s^T B \phi_f \phi_f^T B^T \phi_s & -\sqrt{\rho c^2}\phi_s^T B \phi_f \sqrt{D_f} \\ -\sqrt{\rho c^2}\sqrt{D_f}\phi_f^T B^T \phi_s & D_f \end{bmatrix} \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} \tag{15}$$

where $\xi_s = \eta_s$, and $\xi_f = (\sqrt{\rho c^2}\sqrt{D_f})^{-1}\eta_f$

B. Numerical Simulations

In this section, the vibroacoustical responses are represented in terms of mean-square pressures and velocities [Eq. (16)] for a simple example. This later consists of a rigid cavity $(0.312 \times 0.351 \times 0.14)$ m^3 with only one simply supported elastic face ($E = 72$ GPa, $\nu = 0.3$, and $\rho = 2700$ kg/m^3) of a thickness of $t = 0.0015$ m. This plate is excited by unit punctual force at point (0.039,0.0702,0) m [Fig. 2]:

$$\begin{cases} \langle v^2 \rangle = \frac{1}{2S_s}\int_{S_s} |V_n|^2 \, \mathrm{d}S \\ \langle p^2 \rangle = \frac{1}{2V_f}\int_{V_f} |p|^2 \, \mathrm{d}V \end{cases} \tag{16}$$

The response of the coupled system is performed using the following methods: 1) the DM, given by Eq. (7); 2) the classical modal method (CMM), given by Eq. (12); and 3) the symmetrization of the modal method (SMM) proposed by Sandberg [15], given by Eqs. (14) and (15).

The plate is discretized into (40×50) discrete Kirchhoff quadrilateral (DKQ) plate elements, whereas the cavity is divided into $(40 \times 50 \times 4)$ eight-node brick elements. Hence, the mesh consists of 10,455 DOFs for cavity pressure and 6273 DOFs for the structure displacement and rotation. This implies the resolution of a system of (16728×16728) in case of the DM. It is to be emphasized that the models in the second and the third method use 50 structural modes ($f^s_{\text{mode}=50} = 2456$ Hz) and 50 cavity modes leading to a reduced system of (100×100). Two different cases are considered to compare these methods. In the first one, the cavity is filled with air ($\rho = 1.21$ kg/m^3, $c = 343$ m/s, and $f^f_{\text{mode}=50} = 2722$ Hz) while, in the second case, we use water ($\rho = 1000$ kg/m^3, $c = 1500$ m/s, and $f^f_{\text{mode}=50} = 11,904$ Hz) [Fig. 2]. Figures are given for a frequency range of [1–700] with a step of 1 Hz.

Figure 3 represents the mean-square velocity of the structure, while Fig. 4 gives the mean-square pressure in the cavity when it is filled with air. The different methods agree well with each other.

These results are calculated using the first 50 modes for the structure ($f^s_{\text{mode}=50} = 2456$ Hz) and 50 modes for the fluid ($f^f_{\text{mode}=50} =$ 2722 Hz).

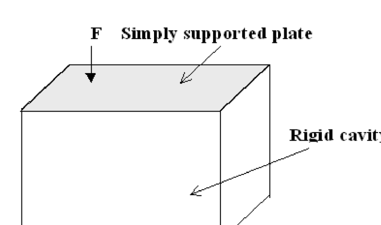

Fig. 2 Plate-backed cavity vibroacoustic problem.

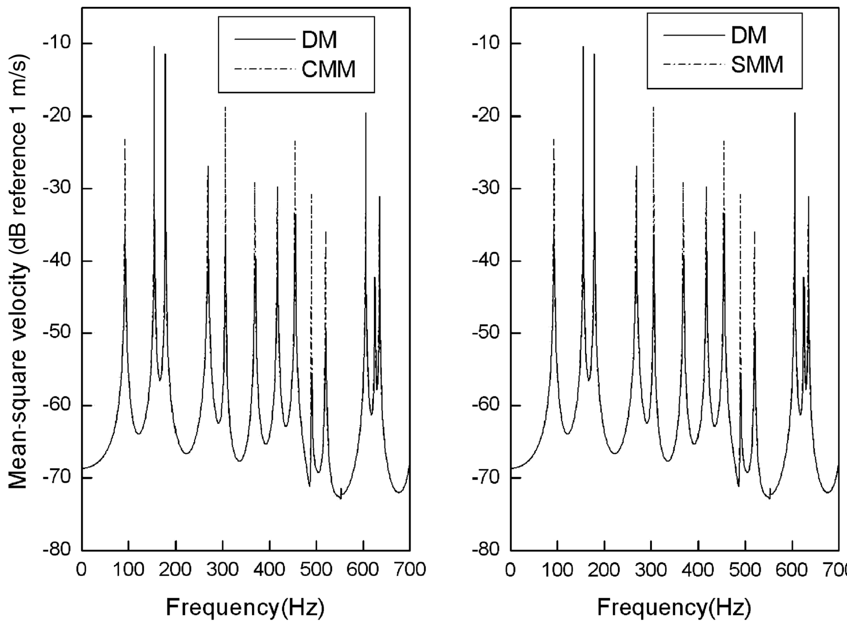

Fig. 3 Mean-square velocity of the plate coupled to a cavity filled with air.

When the cavity is filled with water, the response is performed for the following range of frequency: 1 to 700 Hz. The modal analysis, the DM, and the symmetrization procedure lead to different results [Figs. 5 and 6], even with a maximal frequency of 11,904 Hz for the cavity and 2456 Hz for the structure, which are greater than the maximal frequency of the considered spectrum (700 Hz). In fact, the interaction of the structure with a heavy fluid like water leads to a strong coupling between the structure and cavity modes, including the truncated ones. Consequently, the application of the modal analysis can give wrong physical results for strong coupling problems, since the accuracy is not guaranteed. Hence, it is necessary to remedy the observed poor precision of the method for liquids.

The symmetrization of the modal matrix with the PSC [22] is a solution presented in the next section.

IV. Symmetrization of Modal System with Pseudostatic Correction

A. Pseudostatic Correction

The PSC [22] is shortly described in this section. The structure and cavity truncated modes are ϕ_{fn} and ϕ_{sn}, respectively:

$$\begin{cases} u = \phi_s\xi_s + \phi_{sn}\xi_{sn} = \phi_s\xi_s + u_n \\ p = \phi_f\xi_f + \phi_{fn}\xi_{fn} = \phi_f\xi_f + p_n \end{cases} \tag{17}$$

where u_n and p_n represent, respectively, the displacement and the pressure due the truncated modes.

The relation of orthogonality is applied to the truncated modes by assuming that their contribution is only static (structural kinetic and fluid compressibility energies are neglected):

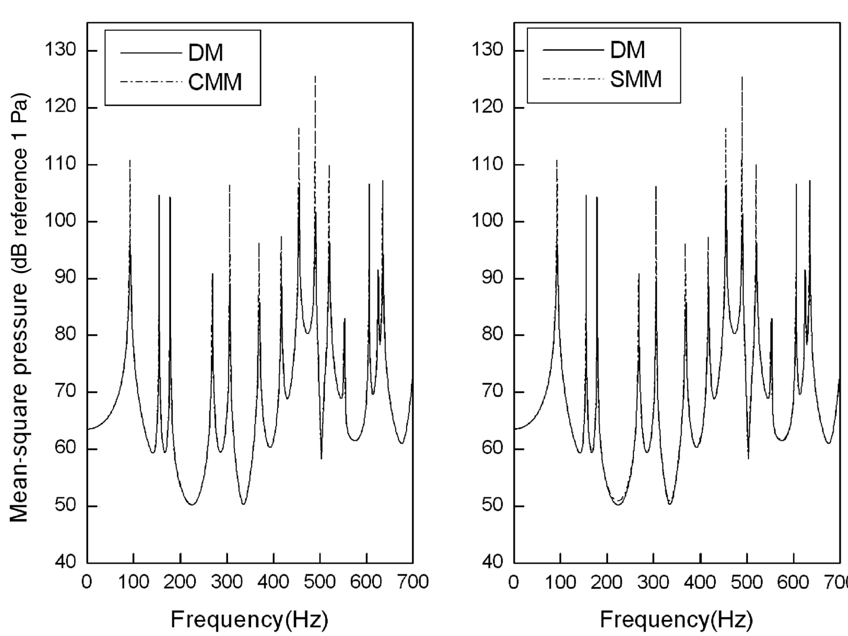

Fig. 4 Mean-square pressure of the cavity filled with air.

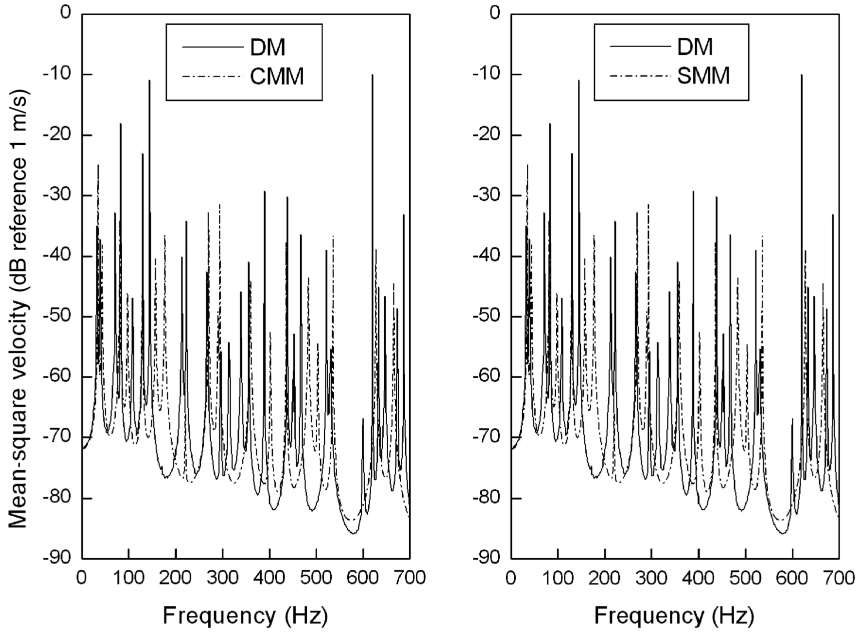

Fig. 5 Mean-square velocity of the plate coupled to a cavity filled with water.

$$\begin{cases} D_{sn}\xi_{sn} \approx \phi_{sn}^T(F_s + Bp) \\ D_{fn}\xi_{fn} \approx \phi_{fn}^T(F_f + \omega^2 B^T u) \end{cases} \tag{18}$$

For the correction of a first order, the neglected modes can be expressed as the following:

$$\begin{cases} u_n = (K_s^{-1} - \phi_s D_s^{-1}\phi_s^T)(F_s + B\phi_f\xi_f) = K_{sn}^{-1}(F_s + B\phi_f\xi_f) \\ p_n = (K_f^{-1} - \phi_f D_f^{-1}\phi_f^T)(F_f + \omega^2 B^T\phi_s\xi_s) = K_{fn}^{-1}(F_f + \omega^2 B^T\phi_s\xi_s) \end{cases} \tag{19}$$

Consequently, by substituting Eq. (19) in Eq. (17), the structural displacement and the fluid pressure can be expressed by

$$\begin{cases} u = \phi_s\xi_s + K_{sn}^{-1}F_s + F_{sadd}^T\xi_f \\ p = \phi_f\xi_f + K_{fn}^{-1}F_f + \omega^2 F_{fadd}^T\xi_s \end{cases} \tag{20}$$

Combining Eqs. (7) and (20), with consideration of orthogonality relations, lead to the following linear system:

$$\begin{bmatrix} D_s - \omega^2(I_s + M_{sadd}) & -\phi_s^T B\phi_f \\ -\omega^2\phi_f^T B^T\phi_s & D_f - \omega^2(I_f + M_{fadd}) \end{bmatrix} \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{Bmatrix} \phi_s^T F_s + F_{fadd}F_f \\ \phi_f^T F_f + \omega^2 F_{sadd}F_s \end{Bmatrix} \tag{21}$$

where M_{sadd} and $F_{fadd}F_f$ represent, respectively, the added mass and load due to the neglected cavity modes, and M_{fadd} and $F_{sadd}F_s$ represent, respectively, the added compressibility and load due to the neglected structure modes:

$$M_{sadd} = \phi_s^T B K_{fn}^{-1} B^T \phi_s; \qquad F_{sadd} = \phi_f^T B^T K_{sn}^{-1}$$
$$M_{fadd} = \phi_f^T B^T K_{sn}^{-1} B \phi_f; \qquad F_{fadd} = \phi_s^T B K_{fn}^{-1}$$

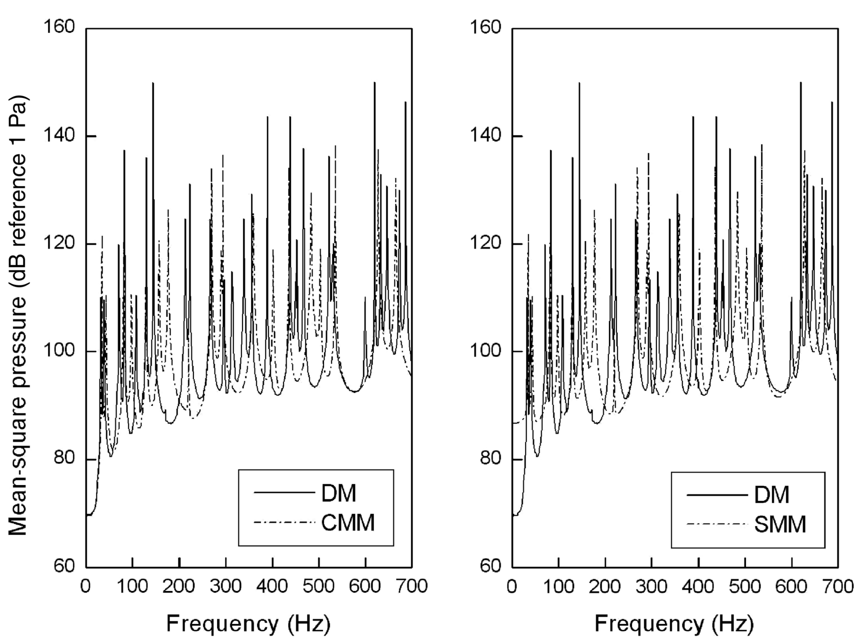

Fig. 6 Mean-square pressure of the cavity filled with water.

In Eq. (19), both K_f and K_s are assumed to be positive definite matrices and, consequently, invertible. Otherwise, a constraint must be applied to these matrices to eliminate rigid modes [24]. More details on PSC can be found in [22].

B. Projection on Coupled Modal Basis

Equation (21) can be written as the following:

$$\begin{bmatrix} D_s & -C \\ 0 & D_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} - \omega^2 \begin{bmatrix} A_s & 0 \\ C^T & A_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{Bmatrix} S_t \\ F_t \end{Bmatrix} \tag{22}$$

where $A_s = I_s + M_{s\mathrm{add}}, A_f = I_f + M_{f\mathrm{add}}, C = \phi_f^T B \phi_f, S_t = \phi_s^T F_s + F_{f\mathrm{add}} F_f$, and $F_t = \phi_f^T F_f + \omega^2 F_{s\mathrm{add}} F_s$.

1. Right Eigenvectors

Equation (22) can be used to calculate the right eigenvectors. The system matrix in Eq. (23), which is almost symmetric, can be obtained by multiplying Eq. (22) on the left by the inverse of the mass matrix:

$$-\omega^2 \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} + \begin{bmatrix} A_s^{-1} D_s & -A_s^{-1} C \\ -A_f^{-1} C^T A_s^{-1} D_s & A_f^{-1} C^T A_s^{-1} C + A_f^{-1} D_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{Bmatrix} A_s^{-1} S_t \\ -A_f^{-1} C^T A_s^{-1} S_t + A_f^{-1} F_t \end{Bmatrix} \tag{23}$$

Using the base given by Eq. (24) permits us to transform Eq. (23) into a symmetric system, presented in Eq. (25):

$$\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{bmatrix} (\sqrt{D_s})^{-1} & 0 \\ 0 & (\sqrt{A_f})^{-1} \end{bmatrix}\begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = S^R \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} \tag{24}$$

$$-\omega^2 \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} + \begin{bmatrix} \sqrt{D_s} A_f^{-1} \sqrt{D_s} & -\sqrt{D_s} A_s^{-1} C (\sqrt{A_f})^{-1} \\ -(\sqrt{A_f})^{-1} C^T A_s^{-1} \sqrt{D_s} & (\sqrt{A_f})^{-1} (C^T A_s^{-1} C + D_f)(\sqrt{A_f})^{-1} \end{bmatrix} \times \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = \begin{Bmatrix} \sqrt{D_s} A_s^{-1} S_t \\ -(\sqrt{A_f})^{-1} (C^T A_s^{-1} S_t + F_t) \end{Bmatrix} \tag{25}$$

K_η and K_ξ are the matrices in the η and ξ systems, respectively. V_η^R and V_ξ^R are the right eigenvectors of the systems in η and ξ, respectively. According to the multiple steps done previously, the following form can be written: $K_\xi S^R V_\eta^R = \lambda S^R V_\eta^R$. Since the eigenvalues do not change, the eigenvectors V_ξ are equivalent to those of $S^R V_\eta^R$. This result permits the deduction of the relation in Eq. (26) between the right vectors V^R of the original system and those of the symmetric eigenvalue problem V_η^R:

$$V^R = \begin{bmatrix} (\sqrt{D_s})^{-1} & 0 \\ 0 & (\sqrt{A_f})^{-1} \end{bmatrix} V_\eta^R = S^R V_\eta^R \tag{26}$$

2. Left Eigenvectors

The left eigenvectors of the initial problem are solutions of Eq. (27):

$$\begin{bmatrix} D_s & 0 \\ -C^T & D_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \lambda \begin{bmatrix} A_s & C \\ 0 & A_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} \tag{27}$$

This system can be written as an almost symmetric one as the following:

$$\begin{bmatrix} A_s^{-1} D_s + A_s^{-1} C A_f^{-1} C^T & -A_s^{-1} C A_f^{-1} D_f \\ -A_f^{-1} C^T & A_f^{-1} D_f \end{bmatrix}\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \lambda \begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} \tag{28}$$

The scaling in Eq. (29) is used to make Eq. (28) symmetric:

$$\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = \begin{bmatrix} (\sqrt{A_s})^{-1} & 0 \\ 0 & (\sqrt{D_f})^{-1} \end{bmatrix}\begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = s^L \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} \tag{29}$$

Consequently, the final symmetric eigenvalue problem is given by:

$$\begin{bmatrix} A_s^{-1/2} (D_s + C A_f^{-1} C^T) A_s^{-1/2} & -A_s^{-1/2} C A_f^{-1} \sqrt{D_f} \\ -\sqrt{D_f} A_f^{-1} C^T A_s^{-1/2} & \sqrt{D_f} A_f^{-1} \sqrt{D_f} \end{bmatrix}\begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} = \lambda \begin{Bmatrix} \eta_s \\ \eta_f \end{Bmatrix} \tag{30}$$

And the eigenvectors of the original problem are given by

$$V^L = \begin{bmatrix} (\sqrt{A_s})^{-1} & 0 \\ 0 & (\sqrt{D_f})^{-1} \end{bmatrix} V_\eta^L = S^L V_\eta^L \tag{31}$$

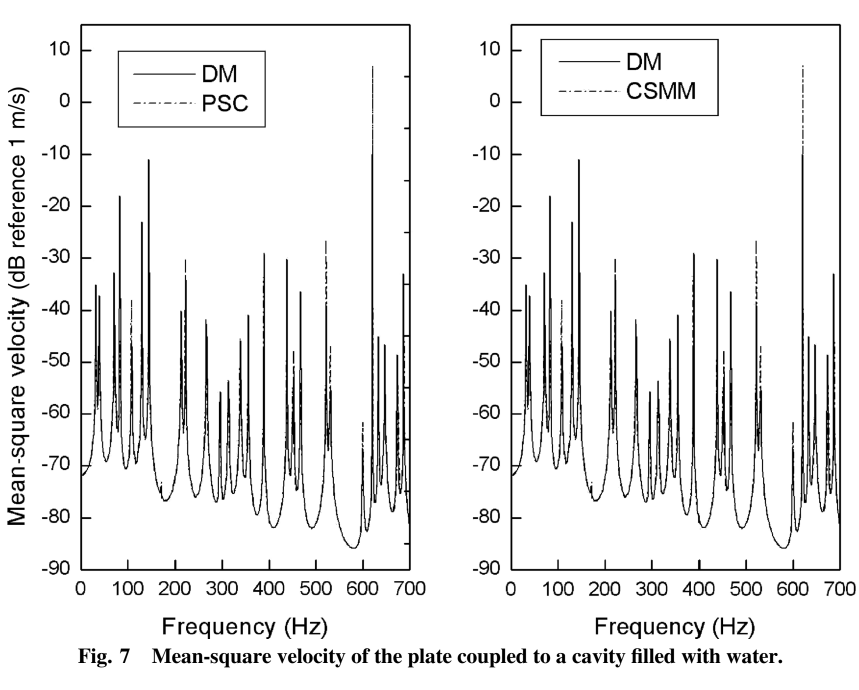

Fig. 7 Mean-square velocity of the plate coupled to a cavity filled with water.

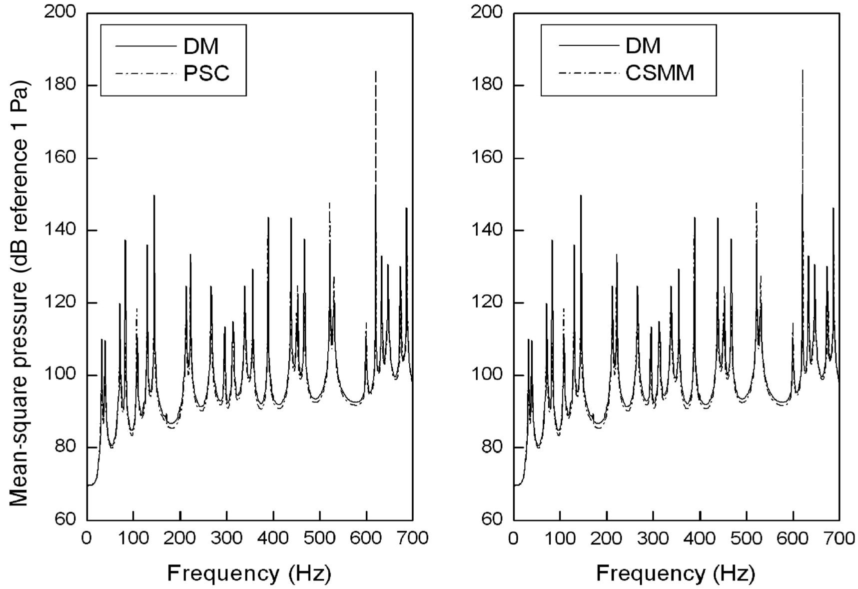

Fig. 8 Mean-square pressure of the cavity filled with water.

3. System Diagonalization

The original system given by Eq. (22) can be written in the following condensed form:

$$(K_m - \omega^2 M_m)\xi = F_{sf} \tag{32}$$

By multiplying Eq. (32) from the left by S_L, with the consideration of$\xi = S^R \eta$, the system takes the following form:

$$(\Gamma_K - \omega^2 \Gamma_M)\eta = S^L F_{sf} \tag{33}$$

where $\Gamma_K = S^L K_m S^R$, and $\Gamma_M = S^L M_m S^R$. Multiplying Eq. (33) from the left by $(V_\eta^L)^T$ and considering the following scaling $(\eta = V_\eta^R \zeta)$ allows us to obtain the diagonal system:

$$(\Omega - \omega^2 I)\zeta = (V_\eta^L)^T S^L F \Rightarrow (\Omega - \omega^2 I)\zeta = (V^L)^T F_{sf} \tag{34}$$

where $\Omega = (V_\eta^L)^T \Gamma_K V_\eta^R$ is a diagonal matrix constituted from the eigenvalues of the coupled system, and the eigenvectors are defined as $(V_\eta^L)^T \Gamma_M V_\eta^R = I$.

The introduction of the different last transformations ($\xi = S^R \eta = S^R V_\eta^R \zeta = V^R \zeta$) permits us to deduce the final solution of the problem:

$$\begin{Bmatrix} \xi_s \\ \xi_f \end{Bmatrix} = V^R \frac{(V^L)^T F_{sf}}{(\Omega - \omega^2 I)} \tag{35}$$

where $F_{st}^T = [\, S_t^T \quad F_t^T \,]$ is the total load. This last equation gives the modal pressure and displacement of the coupled system calculated by taking the static correction into account. The physical unknown variables are obtained by substituting the value of ξ_s and ξ_f in Eq. (20).

One can see that the multifrequency problem analysis consists of an inversion of a diagonal matrix for each spectrum frequency. Hence, once the left and right eigenvectors are calculated, the solution of the problem is obtained by simple matrix multiplications. Another advantage of this method is that the resonance frequencies and the mode shape of the coupled system can be deduced, which gives the vibroacoustic behavior of this system in its totality.

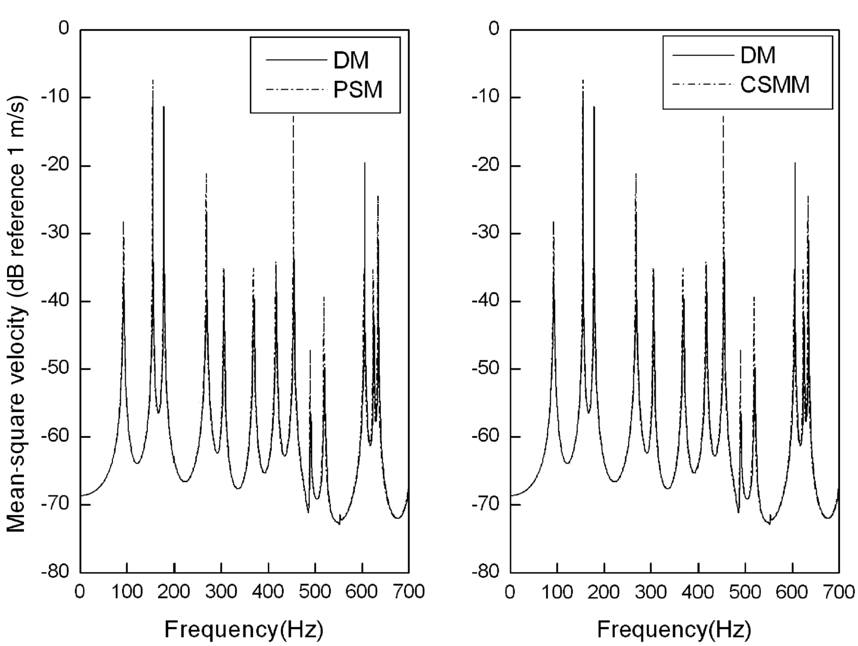

Fig. 9 Mean-square velocity of the plate coupled to a cavity filled with air.

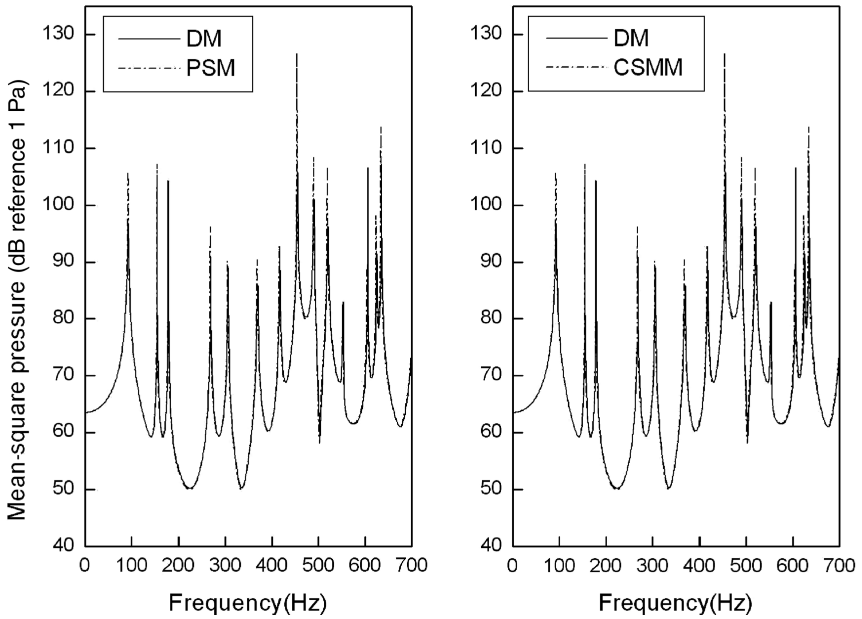

Fig. 10 Mean-square pressure of the cavity filled with air.

C. Validation of the Method

1. Plate-Backed Box Cavity

The example studied in Sec. III is considered. The vibroacoustic response is calculated using the three following methods: 1) the DM, 2) the corrected symmetric modal method (CSMM), and 3) PSC.

Figure 7 shows the mean-square velocity computed by using the two last methods. Good agreement is observed between these methods. The comparisons of Fig. 5 with Fig. 7 and Fig. 6 with Fig. 8 show the improvement of the accuracy by using the static correction. These plots show the accuracy of the present method. The CPU efficiency of the methods is as follows: the DM is 6 days, the CMM is 9.20 s, the SMM is 64.4 s, PSC is 2427 s, and the CSMM is 1146 s. The method presented in this paper appears twice as fast as the PSC, and it is noticeably faster than the DM. Compared to the SMM, the added time in the CSMM to compute the pseudostatic correction is significant for this large problem. This time is decreasing when smaller meshes are studied. This CPU time is considered for the whole frequency range (700 frequencies). It includes all intermediate computations required by each method, except the stiffness, the mass, and the coupling matrices, which are common to all methods.

Finally, the first case consisting of a cavity filled with air is solved by the new method. Both the mean-square pressure and the velocity are still in good agreement with the DM [Figs. 9 and 10]. This last result proves that the present method can be successfully used for weak coupling.

2. Plate-Backed Cylinder Cavity

The test case presented in this section is a rigid cylinder with one elastic steel plate end, while the second one has a spherical shape. The plate, of a thickness of $t = 0.00122$ m, has the following mechanical properties: $E = 210$ GPa, $\nu = 0.3$, and $\rho = 7800\ \mathrm{kg/m^3}$. The cylinder dimensions are represented on Fig. 11. The cavity is filled with water and excited uniformly from its plate end by a unit force.

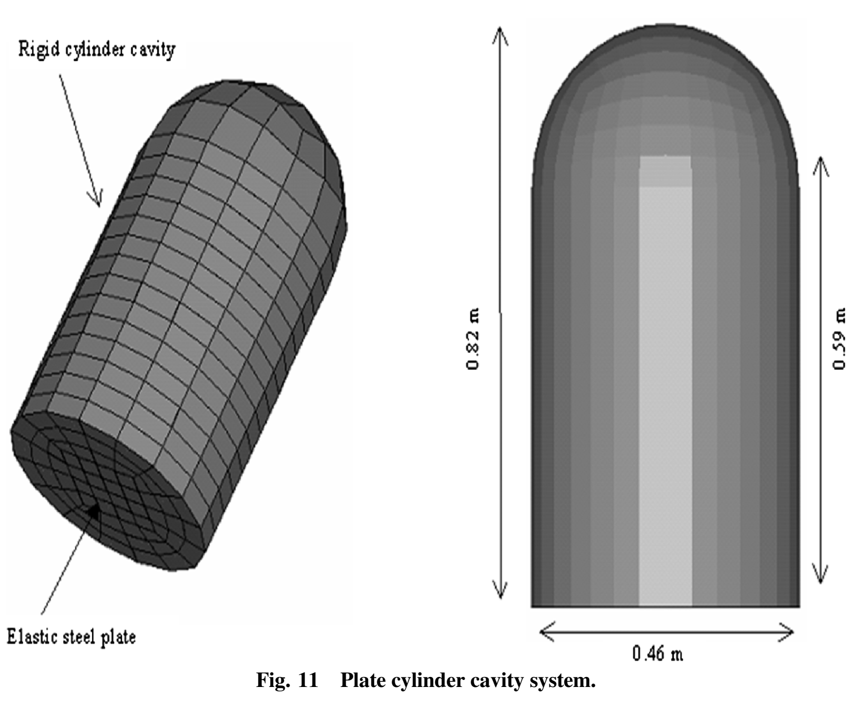

Fig. 11 Plate cylinder cavity system.

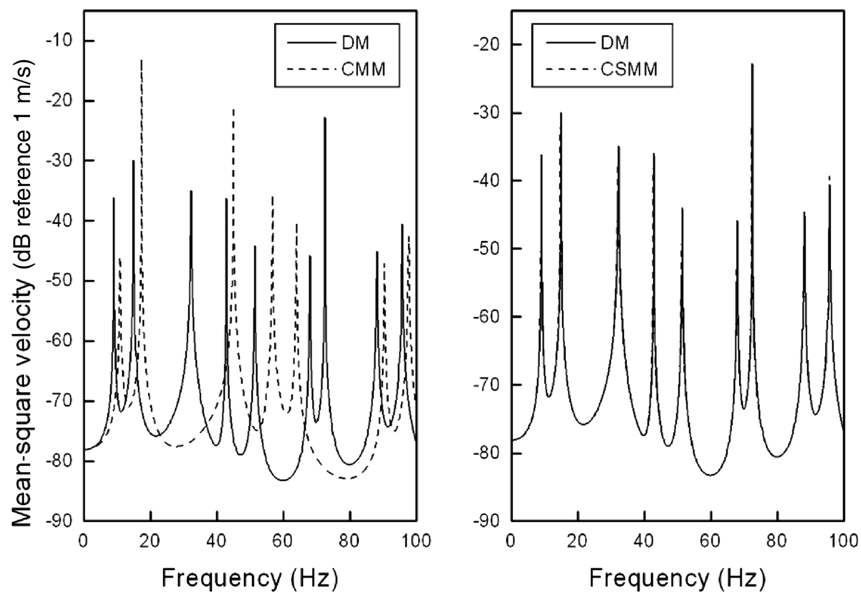

Fig. 12 Mean-square velocity of the plate coupled to the cylinder filled with water.

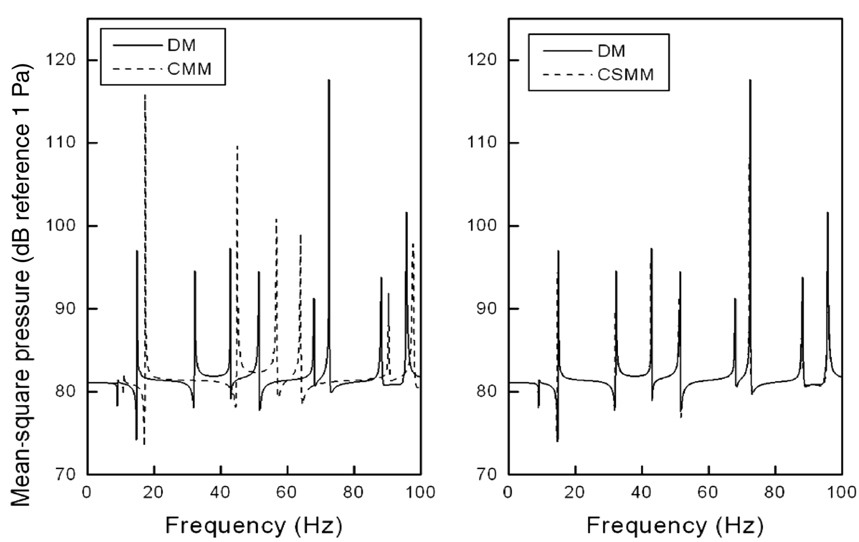

Fig. 13 Mean-square pressure of the cylindrical cavity filled with water.

The plate is discretized using 64 DKQ plate elements, whereas the interior fluid is discretized using 1072 eight-node brick isoparametric elements. The total node number is 1264.

Figures 12 and 13 show the plate mean-square velocity and the cavity mean-square pressure, respectively. Their spectra are plotted for 500 frequencies over a frequency range of [0.2,100] Hz with a step of 0.2 Hz. Fifty plate modes ($f_{s(\mathrm{mode}=50)} = 888$ Hz) and 50 cylinder modes ($f_{f(\mathrm{mode}=50)} = 6201$ Hz) are considered in the modal analysis.

The CMM fails to predict vibroacoustic responses, while there is a full agreement between the DM and the CSMM. It proves the ability of the CSMM to predict the response of the coupling system.

As given in Sec. IV.C.1, estimations of the CPU time for 500 frequencies of [0.2,100] Hz with a step of 0.2 Hz are as follows: the DM is 5113.6 s, the CMM is 59.50 s, and the CSMM is 10.96 s. An important speed up is performed by the CSMM in comparison with the DM and the CMM.

V. Conclusions

This paper deals with a vibroacoustic analysis based on rigid cavity and in-vacuum structure modes to simulate strong structure–cavity interaction, namely, when the interior fluid is a liquid. The eigenvalue system obtained from this method is nonsymmetric. Consequently, the classical efficient algorithms computing eigenvalues and eigenvectors cannot be used. In addition, it does not give any information about the frequencies and the modes of the coupling system.

Among various matrix symmetrization procedures of the pressure-displacement formulation, one possibility is to start from the right and left eigenvectors of the initial unsymmetric matrix system. Although it is great at saving time, this technique can be sometimes dangerous because of the truncation of certain modes susceptible to be required for the coupling.

The symmetrization procedure of the corrected eigenvalue problem using PSC accurately analyzes a vibroacoustic problem. The method not only gives accurate frequencies, modes, and responses, even for the strong coupling problem, but it also proves to be very CPU efficient in comparison with the traditional approaches.

References

[1] Howard, C. Q., Hansen, C. H., and Zander, A., "Vibro-Acoustic Noise Control Treatments for Payload Bays of Launch Vehicles: Discrete to Fuzzy Solutions," *Applied Acoustics*, Vol. 66, No. 11, 2005, pp. 1235–1261.

doi:10.1016/j.apacoust.2005.04.009

[2] Stavrinidis, C., Witting, M., and Klein, M., "Advancements in Vibroacoustic Evaluation of Satellite Structures," *Acta Astronautica*, Vol. 48, No. 4, 2001, pp. 203–210. doi:10.1016/S0094-5765(00)00123-5

[3] Pirk, R., Desmet, W., Pluymers, B., Sas, P., and Goes, L. C. S., "Vibro-Acoustic Analysis of the Brazilian Vehicle Satellite Launcher (VLS) Fairing," *International Conference on Noise and Vibration Engineering, Proceedings of ISMA2002*, Vol. 5, Katholieke Univ. Leuven, Belgium, 16–18 Sept. 2002, pp. 2075–2084.

[4] Pritchard, J., Buehrle, R., Pappa, R., and Grosveld, F., "Comparison of Modal Analysis Methods Applied to a Vibro-Acoustic Test," *Conference on Structural Dynamics No. 20, SPIE Proceedings*, Vol. 4753, No. 2, Society of Photo-Optical Instrumentation Engineers, Bellingham, WA, 4–7 Feb. 2002.

[5] Jollet, D., Giuliano, M., Lodereau, P., and Van Der Laan, T., "Vibroacoustic Study of Scientific Satellites Relying on a Time Domain FEM/BEM Method," *Proceedings of the European Conference on Spacecraft Structures, Materials and Mechanical Testing 2005* [CD-ROM], ESA SP 581, Noodwijk, The Netherlands, 10–12 May 2005.

[6] Alia, A., Souli, M., and Erchiqui, F., "Variational Boundary Element Acoustic Modelling over Mixed Quadrilateral-Triangular Element Meshes," *Communications in Numerical Methods in Engineering*, Vol. 22, 2006, pp. 767–780. doi:10.1002/cnm.848

[7] Alia, A., and Charley, J., "Numerical Simulation of fluid–structure Interaction: Application to Vibroacoustic Problems," *Revue Européenne de Mécanique Numérique*, Vol. 16, Nos. 3–4, 2007, pp. 437–450. doi:10.3166/remn.16.437-450

[8] Alia, A., "Simulation Numérique en Vibroacoustique et en Interaction Fluide Structure," Ph.D. Dissertation, Univ. de Lille, Villeneuve d'ascq, France, 2006.

[9] Alia, A., Djelouah, H., and Bouaoua, N., "Finite Difference Modeling of the Ultrasonic Field Radiated by Circular Transducers," *Journal of Computational Acoustics*, Vol. 12, No. 4, 2004, pp. 475–499. doi:10.1142/S0218396X04002365

[10] Wang, X., and Bathe, K. J., "Displacement/Pressure Based Mixed Finite Element Formulations for Acoustic fluid–structure Interaction Problems," *International Journal for Numerical Methods in Engineering*, Vol. 40, No. 11, 1997, pp. 2001–2017. doi:10.1002/(SICI)1097-0207(19970615)40:11<2001::AID-NME152>3.0.CO;2-W

[11] Ohayon, R., and Soize, C., "Méthodes Numériques Avancées en Vibroacoustique Basses et Moyennes Fréquences," *Revue Européenne des Eléments Finis*, Vol. 8, Nos. 5–6, 1999, pp. 607–637.

[12] Guerich, M., and Hamdi, M. A., "A Numerical Method for Vibro-Acoustic Problems with Incompatible Finite Element Meshes Using *B*-Spline Functions," *Journal of the Acoustical Society of America*, Vol. 105, No. 3, 1999, pp. 1682–1694. doi:10.1121/1.426707

[13] Marburg, S., "Developments in Structural Acoustic Optimisation for Passive Noise Control," *Archives of Computational Methods in Engineering*, Vol. 9, No. 4, 2002, pp. 291–370. doi:10.1007/BF03041465

[14] Treyssède, F., and Elhami, A., "Une Méthode Couplèe de Sous-Structuration Dynamique/Sous Domaines acoustiques pour des Problèmes D'interactions Fluide Structure de Grande Taille," *Proceedings of XIVth Colloque de Vibrations, Chocs et Bruit* [CD-ROM], École Centrale de Lyon, Écully, France, 2004.

[15] Sandberg, G., "A New Strategy for Solving Fluid–Structure Problems," *International Journal for Numerical Methods in Engineering*, Vol. 38, No. 3, 1995, pp. 357–370. doi:10.1002/nme.1620380302

[16] Kim, N. H., Choi, K. K., Dong, J., and Pierre, C., "Design Optimization of Structural Acoustic Problems Using FEM BEM," 44th Structures, Structural Dynamics, and Material Conference AIAA Paper 2003-1630, April 2003.

[17] Kim, N. H., Dong, J., Choi, K. K., Vlahopoulos, N., Ma, Z. A., Castanier, M. P., and Pierre, C., "Design of Sensitivity Analysis of Sequential Structural Acoustic Problems," *Journal of Sound and Vibration*, Vol. 263, No. 3, 2003, pp. 569–591. doi:10.1016/S0022-460X(02)01067-2

[18] Troccas, H. A., Woodcock, R., and Laville, F., "Acoustic Radiation due to Anelastic Impact of a Sphere on a Rectangular Plate," *Journal of the Acoustical Society of America*, Vol. 108, No. 5, 2000, pp. 2197–2202. doi:10.1121/1.1312358

[19] Lambourg, C., Chaigne, A., and Matignon, D., "Time Domain Simulation of Damped Impacted Plates. II Numerical Model and Results," *Journal of Acoustical Society of America*, Vol. 109, No. 4, 2001, pp. 1433–1447. doi:10.1121/1.1354201

[20] Schedin, S., Lambourg, C., and Chaigne, A., "Transient Sound Field from Impacted Plates: Comparison Between Numerical Simulations and Experiments," *Journal of Sound and Vibration*, Vol. 221, No. 3, 1999, pp. 471–490. doi:10.1006/jsvi.1998.2004

[21] Morand, H. J. P., and Ohayon, R., *fluid–structure Interaction*, Wiley, Paris, 1995.

[22] Tournour, M., and Atalla, N., "Pseudostatic Corrections for the Forced Vibroacoustic Response of a Structure Cavity System," *Journal of the Acoustical Society of America*, Vol. 107, No. 5, 2000, pp. 2379–2386. doi:10.1121/1.428624

[23] Wu, T. W., *Boundary Element Acoustics: Fundamentals and Computer Codes, Advances in Boundary Elements*, WIT, Southampton, U. K., 2000.

[24] Géradin, M., and Rixen, D., *Théorie des Vibrations: Application à la Dynamique des Structures*, Masson, Paris, 1993.

J. Astley
Associate Editor